

COMBINING CONSTANT PRESSURE AND DECLINING RATE FILTRATION TO MODEL HIGH PRESSURE DEWATERING ROLLS

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ABSTRACT

High Pressure Dewatering Rolls (HPDR) is a novel filtration technology that involves two counter-rotating semi-permeable rolls inside a feed chamber. The rolls are under vacuum, which induces cake formation on the roll surface followed by cake consolidation once the cake emerges from the feed suspension. The rolls are pulled together hydraulically such that cake compression occurs as the cakes meet and travel through the nip. A model is developed to understand the behaviour of compressible suspensions in the HPDR using one-dimensional compressional rheology theory. The model assumes constant pressure cake formation and consolidation whilst under vacuum followed by controlled rate compression through the nip. The compression rate is determined by the geometry of the rolls and decreases to zero at the narrowest point, necessitating explicit modelling of irreversible compression. The model inputs are the material compressibility and permeability, feed slurry concentration, roll dimensions, roll rotational rate, vacuum pressure, and gap. The model is solved numerically to predict the cake solids concentration, throughput, and force on the rolls. An exact similarity solution exists when the feed concentration is less than the gel point and the roll resistance is negligible, which predicts that the solids throughput increases with the square-root of angular velocity and submerged arc length. Overall, these findings contribute to a better understanding of HPDR behaviour and provide insights for optimising performance in practical applications.

KEYWORDS:

Cake Filtration, High Pressure Filtration, Compressibility, Mathematical model

1. Introduction

Solid-liquid separation is used throughout most process industries. Separating liquid from suspended solids in waste suspensions such as mineral tailings slurries or wastewater sludge reduces the volume of waste along with subsequent storage and disposal costs as well as recovering process water and dissolved chemicals. Valuable suspensions such as pharmaceuticals and chemicals require liquid removal to meet product specifications. A broad range of devices are used to perform solid-liquid separation, including screens, thickeners, filters, and centrifuges. However, fine particulate suspensions are typically compressible such that the equilibrium extent of separation is pressure dependent. Achieving high solids contents requires high driving forces via pressure filtration with corresponding high capital and operating costs.

The University of Melbourne has developed a novel technology called High Pressure Dewatering Rolls (HPDR) (Scales et al., 2017; Höfgen et al., 2019; Hassan et al., 2025;

Hassan et al., 2026) with the aim of providing efficient high pressure filtration. This technology utilizes two counterrotating, semi-permeable rolls under vacuum to generate filter cake and drain filtrate. One roll is fixed in position while the other is free to pivot. The rolls are pulled together by hydraulic pressure, which compresses the cake as it passes between the rolls. Further, by using different roll speeds, the device can introduce shear to enhance dewatering. The technology has several benefits over filter presses, including continuous operation, multiple controllable dewatering mechanisms (vacuum pressure, compressive pressure and shear), and no filter cloths.

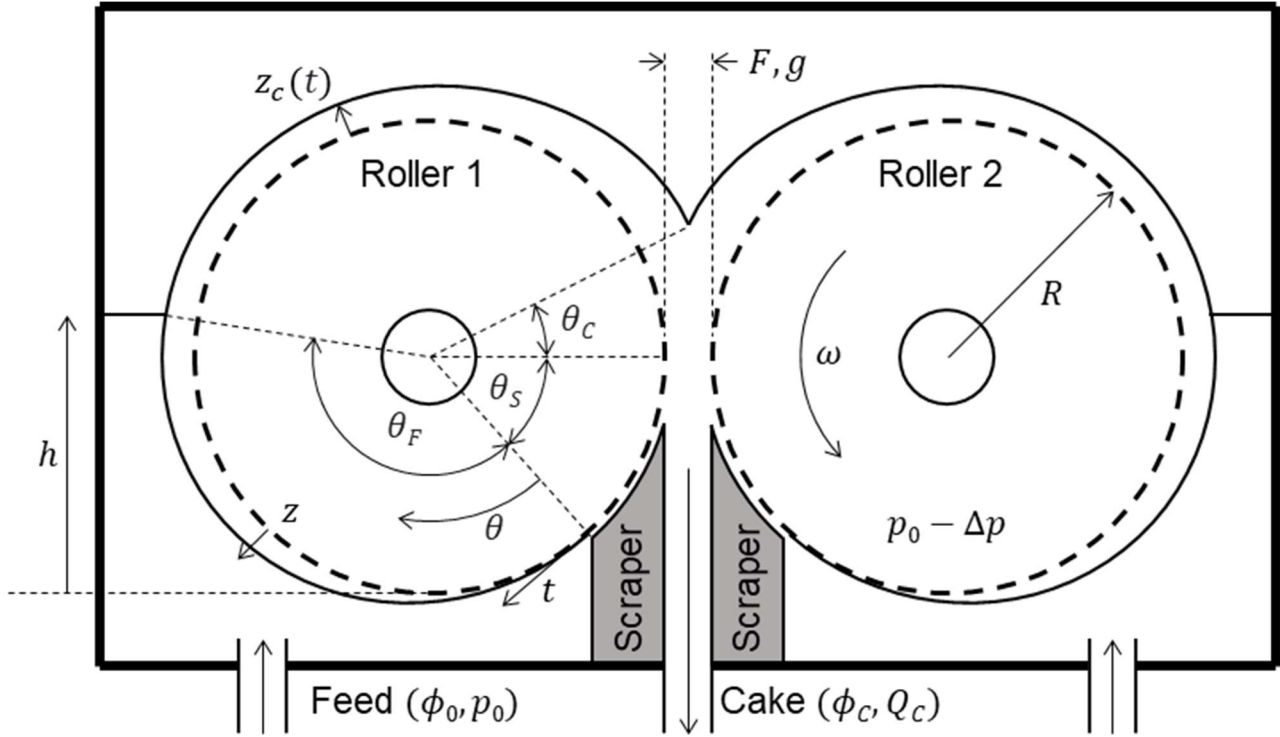


Figure 1: Schematic of HPDR

As illustrated in Figure 1, the HPDR consists of two axially parallel counter-rotating rolls of radius R and length L separated by a gap g . The rolls are pulled together with a force F . The rolls have a semi-permeable surface that nominally allows liquid to pass but not solids – the first HPDR prototype uses a series of discs and shims with an aperture of 0.5 mm (Höfgen et al., 2019) while the second prototype uses a wedge wire screen with an aperture of 30 μm (Hassan et al., 2025). Although the rolls can rotate at different speeds and induce a shear flow, it is assumed in this modelling work that both rolls are rotating at the same angular speed, ω , preserving the symmetry and thereby allowing for modelling of just one roll. The rolls are submerged (depth h , subtended angle θ_F) in the feed slurry with a solid volume fraction of ϕ_0 . The insides of both rolls are held at a vacuum pressure $p_0 - \Delta p$, where p_0 is the ambient pressure. The differential pressure due to vacuum, Δp , causes filter cake to form on the permeable rolls. Cake formation happens until the cake emerges from the feed slurry (θ_F), followed by cake compression and possibly desaturation due to the vacuum pressure. The cakes from the two rolls meet and pass through the nip, undergoing controlled rate cake consolidation given by the HPDR geometry (θ_c). Two scrapers are positioned close to the nip point of the gap (θ_s) to scrape the filter cake from the rolls.

Unfortunately, due to the unique geometry and operating mode of HPDR, there are no existing mathematical descriptions of the process, which would be useful for process design and optimisation. While traditional mathematical models of solid-liquid separation use incompressible descriptions of material behaviour to describe filtration devices, flocculated and ultrafine suspensions, which are intended applications for the HPDR, tend to be compressible such that the rate and equilibrium extent of filtration are pressure dependent. Buscall and White, 1987, presented a phenomenological model for dewatering compressible suspensions, which is one of the common frameworks employed to model filtration in dead-end pressure filters (Landman et al., 1991), vacuum drum and disk filters (Stickland et al., 2011), and filter presses (Stickland et al., 2006; Stickland et al., 2008). The compressional rheology framework (Buscall and White, 1987) assumes that, beyond a critical solid fraction called the gel point ϕ_g , the solid phase (with volume fraction ϕ) forms a continuous network with a physical strength called the compressive yield stress, $p_y(\phi)$. The network breaks and fluid is exuded when the applied pressure exceeds $p_y(\phi)$. The effect on interphase drag force due to interactions with neighbouring particles at the solid phase of volume fraction ϕ is captured via the hindered settling function, $R(\phi)$.

This study presents a 1D model of HPDR for compressible materials without shear (different roll speeds). Building on Stickland et al., 2011's work on vacuum drum and disc filtration, calculations for the cake formation, cake meeting point, and cake consolidation in the HPDR are described. The conservation of mass and momentum equations are solved using the boundary conditions outlined. Model results illustrate the effect of operating parameters, ω , Δp , and ϕ_0 , and geometry parameters, r_r and g for a given material. The 1D model of HPDR allows prediction of the device performance and optimization of the operating conditions and provides a basis for upscaling the design.

2. Theory

The filtration model of HPDR presented in this study assumes that curvature effects can be neglected since the cake is thin compared to the roll diameter. Therefore, the cake formation and compression are considered as one-dimensional using a Cartesian coordinate, z , measured from the roll surfaces. The angular position is denoted by t , where $t = 0$ is the point where the rolls initially contact the slurry inside the chamber. The value of t can be calculated as the ratio of the arc length to the angular velocity at a specific angular position:

$$t = \frac{\theta}{\omega} \quad (1)$$

Therefore, θ_F , θ_C and θ_S map to $t_F = \theta_F/\omega$, $t_C = (\theta_F + \theta_C)/\omega$, and $t_S = (2\pi - \theta_S)/\omega$, respectively.

2.1. Compressional Rheology

The compressional rheology model proposed by Buscall and White, 1987, offers a phenomenological theoretical framework for assessing the dewatering of suspensions by balancing the network pressure and hydrodynamic drag forces between the solid and liquid phases acting on a given volume element of the suspension without taking into account any effects of shear. The framework can be applied to any solid-liquid suspension, as the properties $p_y(\phi)$ and $R(\phi)$ describe the dewatering behaviour of suspensions. The model has been used to describe one-dimensional constant pressure and constant flux piston-driven filtration (Landman et al., 1991; Landman and White, 1994; Landman and White, 1997), as well as vacuum drum and disk filtration (Stickland et al., 2011). Neglecting the effect of

acceleration terms such as gravity and centrifugal force, the transient conservation equations for the momentum of the solid and liquid phases in one dimension (z) are given by Eqs. 2a and 2b:

$$\frac{\phi}{1-\phi} R(\phi)(u_s - u_f) - \frac{\partial p_s}{\partial z} = 0 \quad (2a)$$

$$\frac{\partial p_f}{\partial z} + \frac{\partial p_s}{\partial z} = 0 \quad (2b)$$

Here $u_s - u_f$ is the local velocity of the solids relative to the fluid, and p_f and p_s are the local solid and liquid pressures. Transient conservation equations for the mass for solid and liquid in one dimension (z) are given by Eqs. (3a) and (3b) assuming incompressibility of each phase:

$$\frac{\partial \phi}{\partial t} = \frac{\partial \phi u_s}{\partial z} \quad (3a)$$

$$\frac{\partial(1-\phi)}{\partial t} = \frac{\partial(1-\phi)u_f}{\partial z} \quad (3b)$$

The conservation of suspension volume follows by combining equations 3a and 3b;

$$\phi u_s + (1-\phi)u_f = q(t) \quad (4)$$

where $q(t)$ is the bulk flow, which is equal to the specific flow rate of the filtrate, dv/dt (where v is the specific volume of the filtrate at time t). By combining Eqs. (2) and (4);

$$\frac{\phi}{(1-\phi)^2} R(\phi)(u_s - q) - \frac{\partial p_s}{\partial z} = 0 \quad (5)$$

Eliminating the solid flux gives

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} \left[\frac{(1-\phi)^2}{R(\phi)} \frac{\partial p_s}{\partial z} + \phi q \right] \quad (6)$$

For applications where the pressure is constant or monotonically increasing, there is no need to consider the reversibility (or irreversibility) of consolidation, allowing substitution of $p_y(\phi) = p_s$ into Eq. (6) to give the governing equation of the one-dimensional pressure filtration (Buscall and White, 1987);

$$\frac{\partial \phi}{\partial t} = \frac{\partial}{\partial z} \left[D(\phi) \frac{\partial \phi}{\partial z} + \phi q \right] \quad (7)$$

where

$$D(\phi) = \frac{(1-\phi)^2}{R(\phi)} \frac{dp_y(\phi)}{d\phi} \quad (8)$$

2.2. Constant Pressure Cake Formation

Cake formation occurs when the cake builds up on the surface of the rolls under vacuum pressure until it reaches the nip point or when the two cakes meet, whichever occurs first.

The initial conditions for cake formation are;

$$\phi(z, 0) = \phi_0 \quad (9a)$$

$$v(0) = 0 \quad (9b)$$

$$z_c(0) = 0 \quad (9c)$$

where $z_c(t)$ is the cake height.

The model assumes that $\phi_0 < \phi_g$, so that feed slurry in the chamber is not networked and that there is no network of solids above the cake (the concentration where at which a network forms, $p_y(\phi_g) = 0$); therefore, the top of the cake is at the gel point, ϕ_g :

$$\phi(z_c(t), t) = \phi_g \quad (10)$$

Since the material above the cake ($z > z_c(t)$) remains at the bulk concentration, ϕ_0 , there is a discontinuity at $z_c(t)$. The concentration gradient at the top of the cake is given by integrating the solids conservation Eq. (3a) across the discontinuity at $z_c(t)$ and rearranging:

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=z_c(t)} = -\frac{\phi_g - \phi_0}{D_g} \left(\frac{dz_c}{dt} + q \right) \quad (11)$$

where D_g is $D(\phi_g)$.

The solids pressure at the rolls is the applied pressure minus the pressure drop across the rolls caused by the roll resistance. When it is assumed that the roll resistance is negligible compared to the cake resistance, then the pressure boundary condition at $z = 0$ is simply the solid pressure equal to the applied vacuum pressure, $p_s = \Delta p$. Given that $p_s = p_y(\phi)$, the solid concentration at the surface of the rolls is defined by the applied vacuum pressure, which is constant:

$$\phi(0, t) = \phi_\infty \quad (12)$$

Here ϕ_∞ is the equilibrium value when $\Delta p = p_y(\phi_\infty)$. Assuming that the rolls are impermeable to all solids such that the solids flux is zero, the concentration gradient at the rolls is:

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=0} = -q \frac{\phi_\infty}{D_\infty} \quad (13)$$

where D_∞ is $D(\phi_\infty)$.

Similarity Solution

Analogous to constant cake formation in the dead-end filtration case under constant pressure (Landman and White, 1997), there is a similarity solution for this problem when for $\phi_0 < \phi_g$, regardless of the constitutive equations used to describe $p_y(\phi)$ and $R(\phi)$. The similarity variable, X is defined:

$$X(z, t) = \frac{z}{z_c(t)} \quad (14)$$

such that

$$\phi(z, t) = \begin{cases} \Phi(X) & z \leq z_c(t) \\ \phi_0 & z > z_c(t) \end{cases} \quad (15)$$

The similarity solutions for $z_c(t)$ and $q(t)$ vary with the square-root of t ;

$$z_c(t) = \gamma\sqrt{t} \quad (16a)$$

$$q(t) = \frac{\alpha}{\sqrt{t}} \quad (16b)$$

By changing the variable in the diffusion equation, Eq. (7) and substituting Eq. (15), the partial differential diffusion equation can be transformed into an ordinary differential equation to describe the local concentration as a function of X :

$$\frac{d}{dX} \left(D(\Phi) \frac{d\Phi}{dX} \right) + \left(\frac{\gamma^2}{2} X + \alpha \right) \frac{d\Phi}{dX} = 0 \quad (17)$$

For this solution, the boundary conditions at the roller surface and the top of the cake become:

$$\Phi(0) = \phi_\infty \quad \left. \frac{d\Phi}{dX} \right|_{X=0} = -\alpha \frac{\phi_\infty}{D_\infty} \quad (18a)$$

$$\Phi(1) = \phi_g \quad \left. \frac{d\Phi}{dX} \right|_{X=1} = -\left(\alpha + \frac{\gamma^2}{2} \right) \frac{\phi_g - \phi_0}{D_g} \quad (18b)$$

The implications for this are that the functional form of the cake distribution is constant when the feed is unnetworked, the pressure is constant and the membrane resistance is negligible. Introducing a new variable, η for the similarity variable, X :

$$\eta = \left(\alpha + \frac{\gamma^2}{2} \right) (1 - X) \quad (19)$$

the ordinary differential equation, Eq. (17) can be rewritten as:

$$\frac{d}{d\eta} \left(D(\Phi) \frac{d\Phi}{d\eta} \right) + \left(1 - \frac{\delta^2}{2} \eta \right) \frac{d\Phi}{d\eta} = 0 \quad (20)$$

where δ is:

$$\delta = \left(\frac{\alpha}{\gamma} + \frac{\gamma}{2} \right)^{-1} \quad (21)$$

The boundary equations thus become:

$$\Phi(0) = \phi_{\infty} \quad \frac{d\Phi}{d\eta} \Big|_{\eta=0} = -\alpha \frac{\phi_{\infty}}{D_{\infty}} \quad (22a)$$

$$\Phi(\eta^*) = \phi_g \quad \frac{d\Phi}{d\eta} \Big|_{\eta=\eta^*} = - \left(1 + \frac{\delta^2 \eta^2}{2} \right) \frac{\phi_g - \phi_0}{D_g} \quad (22b)$$

where η^* is:

$$\eta^* = \alpha + \frac{\gamma^2}{2} \quad (23)$$

A numerical solution is used to solve for any arbitrary material properties of $p_y(\phi)$ and $R(\phi)$. Here an iterative Runge–Kutta numerical algorithm (Stickland et al., 2006) is used to solve the governing equation, Eq. (20), subject to the appropriate boundary conditions. By using an initial guess for δ , Eq. (20) is solved from $\eta = 0$ from $\Phi = \phi_{\infty}$ where $\eta = \eta^*$. The δ value is iterated using an interval halving method until the gradient at η^* satisfies the fourth boundary condition, the concentration gradient at the rolls. Once a suitable value for δ is determined, α and γ (and thereby $z_c(t)$ and $q(t)$) are evaluated by rearranging Eqs. (21) and (23):

$$\alpha = \eta^* \left(1 - \frac{\eta^* \delta^2}{2} \right) \quad (24a)$$

$$\gamma = \eta^* \delta \quad (24b)$$

Therefore, during cake formation, for a given vacuum pressure and initial concentration (a constant γ), the cake height, $z_c(t)$, at a fixed angular position only varies with ω . The average solids concentration, $\bar{\phi}$ of the cake at a given location remains constant:

$$\bar{\phi} = \frac{1}{z_c} \int_0^{z_c} \phi(z, t) dz = \frac{1}{\eta^*} \int_0^{\eta^*} \Phi(\eta) d\eta \quad (25)$$

as $(1/\eta^*) \int_0^{\eta^*} \Phi(\eta) d\eta$ in Eq. (25) is constant and does not depend on parameters like ω .

2.3. Cake Contact Point, θ_F

The two cakes that form on the rolls meet in the region $3\pi/2 < \theta - \theta_s < 2\pi$, creating a single cake before passing through the nip point. The exception is if the cake height remains less than half the gap width $z_c(t_s) < g/2$. Just before the two cakes meet, for any angular position, θ , the cake height can be expressed using Eq. (1):

$$z_c(\theta) = \gamma \sqrt{\frac{\theta}{\omega}} \quad (26)$$

Since it is assumed that the two rolls are submerged to the same depth in the feed slurry and are rotating at the same speed, all possible contact points of the two cakes lie in the mid-plane and the problem is symmetrical. The distance to any arbitrary contact point in the symmetry plane can be determined by using the hypotenuse to the symmetry plane from the roll surface, $z_h + r_r$:

$$z_h(\lambda) + r_r = \frac{r_r + \frac{g}{2}}{\cos(\lambda)}; \quad 0 \leq \lambda \leq \frac{\pi}{2} \quad (27)$$

where λ is the angle to the hypotenuse from the horizontal plane.

The angular position at the point of contact, or otherwise at the end of the cake formation, θ_F , can be expressed in terms of z_c and z_h as:

$$z_c(\theta = \theta_F) = z_h(\lambda = 2\pi - \theta_S - \theta_F) \quad (28)$$

Combining Eqs. (26) and (27), θ_F can be determined numerically:

$$\frac{r_r + \frac{g}{2}}{\cos(\theta_F + \theta_S)} - r_r = \gamma \sqrt{\frac{\theta_F}{\omega}} \quad (29)$$

The angle where the cakes meet, θ_F , is a variable that depends on the operating conditions and HPDR geometry. For a given material and fixed scraper location, the point of contact, θ_F , will vary with operating conditions (Δp , ϕ_0 , ω , h) and HPDR geometric properties (g , r_r).

The average solid concentration of the filter cake at the point of contact, ϕ_F , is given by:

$$\phi_F = \frac{1}{z_{c,F}} \int_0^{z_{c,F}} \phi(z, t_F) dz = \frac{1}{\eta^*} \int_0^{\eta^*} \Phi(\eta) d\eta \quad (30)$$

The solid throughput, Q_F , is the amount of solids in the cake formed at the point of contact:

$$Q_F = \omega r_r L z_{c,F} \phi_F \quad (31)$$

By substituting Eq. (30) and simplifying;

$$Q_F = \sqrt{\omega \theta_F} r_r L \delta \int_0^{\eta^*} \Phi(\eta) d\eta \quad (32)$$

Given that γ depends only on the vacuum pressure, initial solid concentration, and material characteristics, ϕ_F is a constant and independent of time or angular speed. For given material at fixed vacuum pressure and initial concentration, the throughput at formation varies with $\sqrt{\omega \theta_F} r_r L$.

2.4. Cake compression under controlled rate

When two formed cakes meet, the cakes compress each other against the rolls expressing filtrate further. Since the formed cake enters this zone, cake compression happens. In this compression zone, $\theta_F < \theta < \theta_C + \theta_F$, the cake height, $z_c(t)$ governs by the geometry:

$$z_c(t) = \frac{r_r + \frac{g}{2}}{\cos(\omega t + \theta_S)} - r_r \quad (33)$$

The negative rate of height variation of filter cake thickness, $dz_c(t)/dt$, is equal to the filtrate flux, $q(t)$:

$$\frac{dz_c}{dt} = -q(t) = \omega \left(r_r + \frac{g}{2} \right) \frac{\sin(\omega t + \theta_F)}{\cos^2(\omega t + \theta_F)} \quad (34)$$

The global solid balance of the cake in the compression region:

$$\phi_C \frac{g}{2} = \int_0^{z_{c,F}} \phi(z, t_F) dz = \int_0^{z_{c,C}} \phi(z, t_C) dz \quad (35)$$

Thus, the average solid concentration at the end of compression, ϕ_C can be expressed interest of the solid amount at the end of formation:

$$\phi_C \frac{g}{2} = \phi_F z_{c,F} \quad (36)$$

The final solid throughput at nip or otherwise at the end of compression, Q_C , is determined via:

$$Q_C = \omega r_r L z_{c,F} \phi_F \quad (37)$$

Combining Eqs. (31) and (37), the solid throughput at the nip can be shown to be the same as the solid throughput at the end of cake formation, $Q_C = Q_F$. The initial conditions for the compression stage are the final conditions of the formation stage, $\phi(z, t_F)$ and $z_c(t_F)$.

Unlike the cake formation stage, the pressure at the rolls changes due to the time-dependent compressional pressure, $\Delta p_c(t)$:

$$p_y(\phi(0, t)) = \Delta p + \Delta p_c(t) \quad (38)$$

As a result, the solids concentration at the rolls varies with time during compression.

The solids velocity at the bottom of the cake equals or otherwise at the rolls is zero, assuming that rolls reject all solids $z(0, t)$:

$$\left. \frac{\partial \phi}{\partial z} \right|_{z=0} = -q \frac{\phi(0, t)}{D(\phi(0, t))} \quad (39)$$

Unlike the piston-driven case or cake formation stage, the boundary condition of the mid-plane is undefined for HPDR, such that a discontinuity may exist. Three possible boundary conditions can be considered assuming the concentration is fixed ($\phi(z_c, t) = 0$), the concentration gradient is zero ($\partial\phi/\partial z|_{(z_c = 0)}$) or the conservation of solids Eq. (35) holds. For this study third condition is checked.

During compression, the velocity at the top of the cake equals the compression rate as the overall volume is conserved:

$$\frac{dz_c}{dt} = -q(t) \quad (40)$$

The governing diffusion equation, Eq. (7) for controlled rate compression, requires a numerical solution. Thus Eq. (7) is simplified to two coupled first-order ordinary differential equations using a backward difference approximation in time, Δt . The concentration gradient at a given time is:

$$\frac{d\phi}{dz} = \frac{\phi}{D(\phi)}(u_s - q) \quad (41)$$

The solids flux gradient is:

$$\frac{d\phi u_s}{dz} = \frac{\phi(z) - \phi^<(z)}{\Delta t} \quad (42)$$

where $\phi^<(z)$ is the value of $\phi(z)$ at the same z value at the previous time step.

The cumulative solids volume fraction, I_ϕ is:

$$\frac{dI_\phi}{dz} = \phi \quad (43)$$

The volume fraction distribution during the cake compression is estimated at progressive steps of $\phi(z = 0)$ by solving Eqs. (41) to (43) using Runge-Kutta numerical technique from the roll surface to the top of the cake height for successive estimates of Δt until the conservation of solids (Eq. (35)) is satisfied. The accuracy of the numerical technique is indicated by the volume fraction gradient at z_c .

3. Materials and Method

Material properties for $p_y(\phi)$ and $R(\phi)$ are based on a model tailing suspension:

$$p_y(\phi) = 100 \left[\left(\frac{\phi}{\phi_g} \right)^5 - 1 \right] \text{ Pa; } \phi_g = 0.15 \text{ v/v} \quad (44a)$$

$$R(\phi) = 10^8 (1 - \phi)^{-5.5} \text{ Pa.s/m}^2 \quad (44b)$$

The 1D model discussed is implemented in Mathematica® to solve the problem numerically. The model predicts the solid throughput and concentrations for different operating conditions

and HPDR roll sizes. The roll length and scraper angle was the same in all simulations ($L = 1$ m and $\theta_S = \pi/4$).

4. Results

4.1. Filter cake solid distribution and cake height

The solution of the numerical model for $\omega = 2\pi/60$ rad/s (1 rpm), $\Delta p = 10$ kPa, $\phi_0 = 0.1$ v/v, $g = 0.03$ m and $r_r = 0.5$ m is presented as a distribution plot in Figure 2 and as a contour plot in Figure 3. The cake height as a function of time is shown in Figure 4.

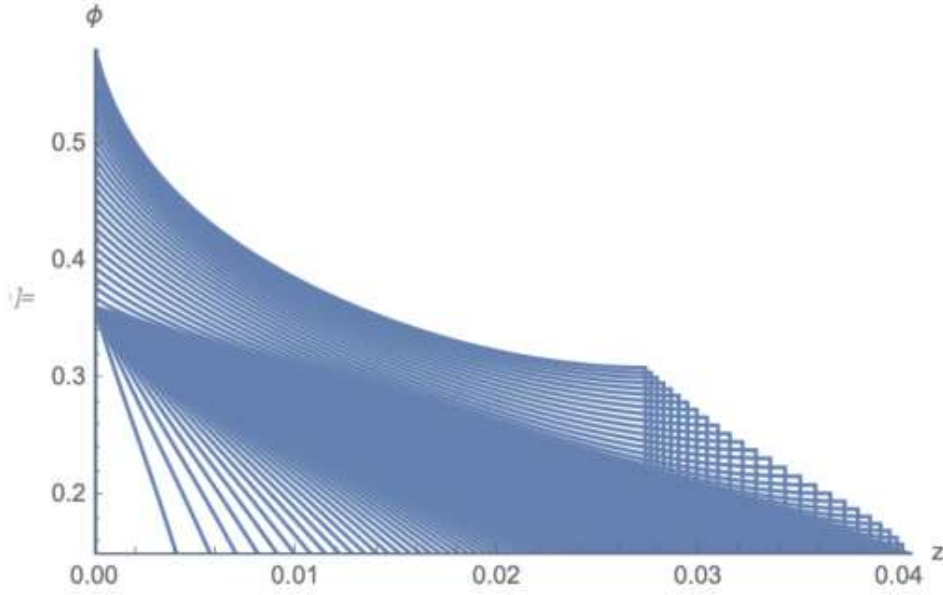


Figure 2: Solid distribution profile ($\Delta P = 10$ kPa, $\omega = 2\pi/60$ rad/s, $\phi_0 = 0.1$ v/v, $g = 0.03$ m and $\theta_S = \pi/4$ rad).

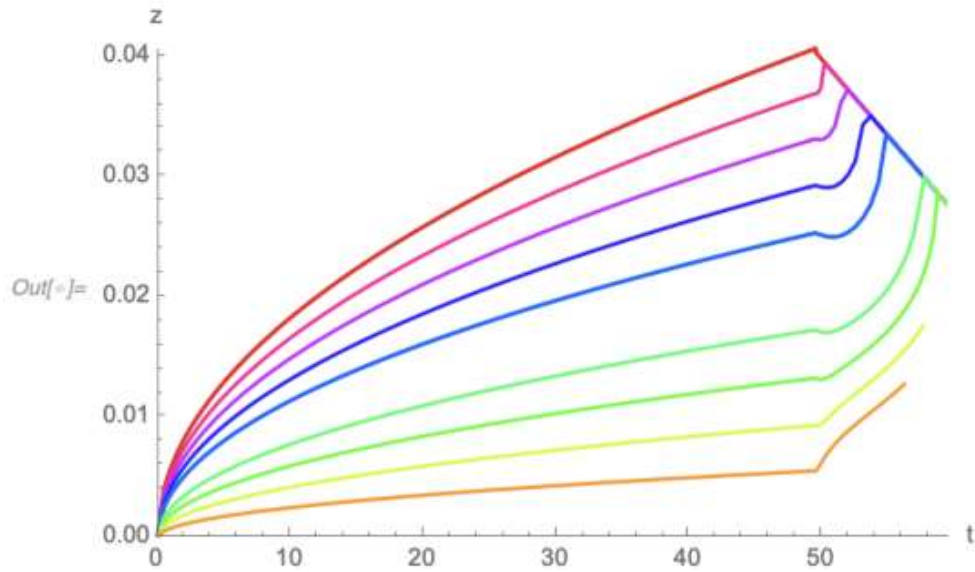


Figure 3: Solid contour plot ($\Delta P = 10$ kPa, $\omega = 2\pi/60$ rad/s, $\phi_0 = 0.1$ v/v, $g = 0.03$ m and $\theta_S = \pi/4$ rad).

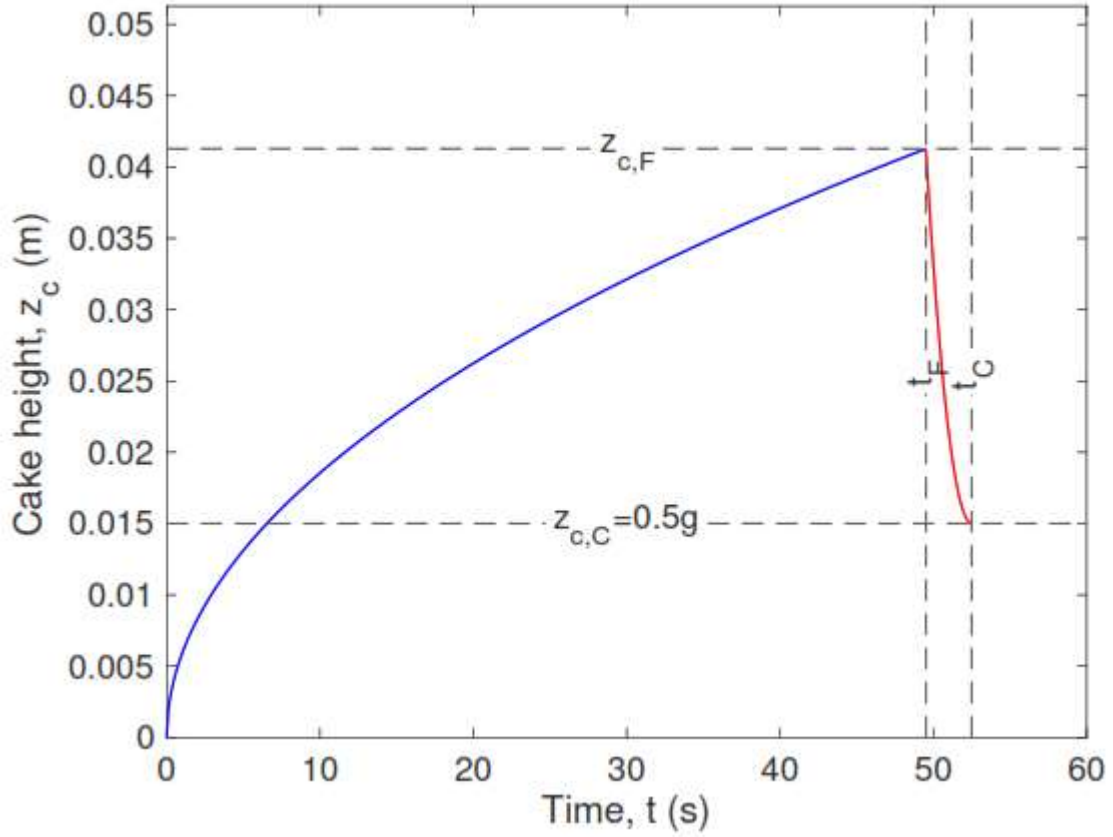


Figure 4: Cake height variation on rolls with time ($\Delta P = 10$ kPa, $\omega = 2\pi/60$ rad/s, $\phi_0 = 0.1$ v/v , $g = 0.03$ m and $\theta_S = \pi/4$ rad).

4.2. θ_F variation with operating conditions

One of the important parameters in HPDR is θ_F as it determines the start of the compression. Figure 5 illustrates the cake meeting point variation with operating conditions and geometry parameters in a range of practicality. With increasing Δp , θ_F decays rapidly within a small range, suggesting the two cakes will meet at a high level of mid-plane. With increasing ω and r_r separately, θ_F increases with a decreasing rate and asymptotes to $2\pi - \theta_S$, suggesting that increasing these parameters will lead to un-compressed cakes. Increasing the gap size between the rolls again results in an increase of θ_F , leading to an increase in the likelihood of getting unfiltered filter cakes. As the vacuum pressure drop (Δp) increases, the formation angle θ_F decreases rapidly within a narrow range. This indicates that the two cakes will meet at a higher mid-plane level, suggesting a more compact filtration from compression. On the other hand, increasing the rotational speed (ω) and the radius (r_r) individually leads to an increase in θ_F with a diminishing rate. Eventually, θ_F approaches $2\pi - \theta_S$, indicating that higher values of these parameters can result in un-compressed cakes. Furthermore, increasing the gap between the rolls causes an increase in θ_F , increasing the likelihood of obtaining un-compressed filter cakes.

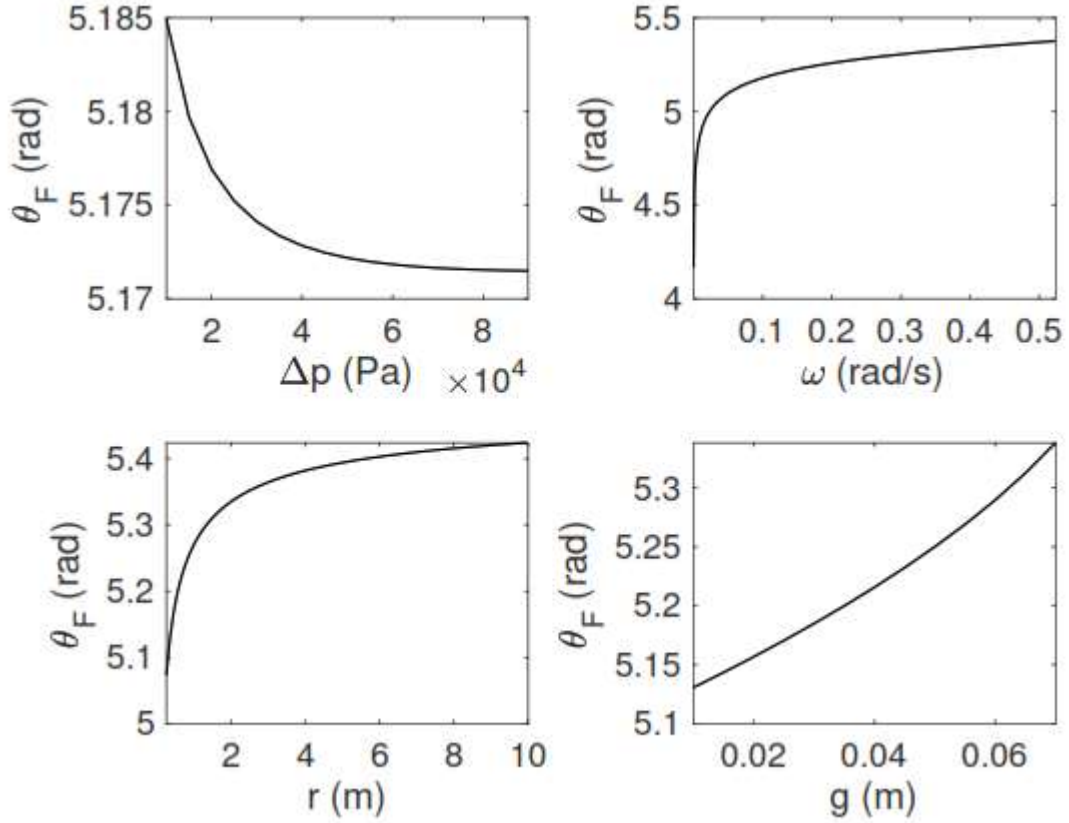


Figure 5: Variation of the angular position where two cakes meeting, θ_F , with vacuum pressure, angular velocity, roll radius, and gap width (while varying a single parameter, fixed parameters are maintained at $\Delta P = 10$ kPa, $\omega = 2\pi/60$ rad/s, $\phi_0 = 0.1$ v/v, $r_r = 0.5$ m, $g = 0.03$ m and $\theta_S = \pi/4$ rad).

4.3. Solid throughput and final cake solid volume fraction

During the HPDR operational stage, two adjustable parameters can be typically used to control the process: angular speed and vacuum pressure for a given tailings suspension. Figure 6 shows the variation of Q and ϕ_C with ω operating at constant vacuum pressure. With increasing ω , ϕ_C decreases, and Q increases. The highest filter cake solids can be seen at the lowest angular velocities. However, it is important to note that for a fixed gap, HPDR cannot function at an angular speed below the minimum threshold value of ω_{min} , provided that $\phi_{FZC,F} \leq \phi_{max}g/2$, where ϕ_{max} represents the maximum solid concentration with a value less than or equal to 1. The determination of ϕ_{max} is crucial, and it should be based on the maximum pressure capacity of the device, denoted as p_{max} . If $\phi_C = \phi_{max}$ at the nip, the solid pressure $p_s(\phi_{max})$ exerted on the rolls becomes p_{max} causing equipment failure. Here we assume $\phi_{max} = 1$, resulting $\omega_{min} = 0.05342$ rad.

Compression of the filter cake occurs only when $z_c(t_{max}) > g/2$, where $t_{max} = (2\pi - \theta_S)/\omega$. As the angular speed ω increases, the likelihood of compression decreases until a certain maximum angular speed (ω_{max}) is reached, at which point the two cakes are expelled from the rolls separately with a final solid concentration of ϕ_F . However, at lower angular speeds, compression occurs, resulting in a higher solids content ($\phi_C > \phi_F$) in the filter cake since $z_{c,F} > z_{c,C}$ ($= g/2$), as determined by the solid balance equation (36). For the considered case,

$\omega_{max} = 0.8408$ rad. For $\omega_{min} < \omega < \omega_{max}$, Q varies with the square root of $\omega\theta_F$, while the solid concentration of the filter cake varies with $\sqrt{\theta_F/\omega}$.

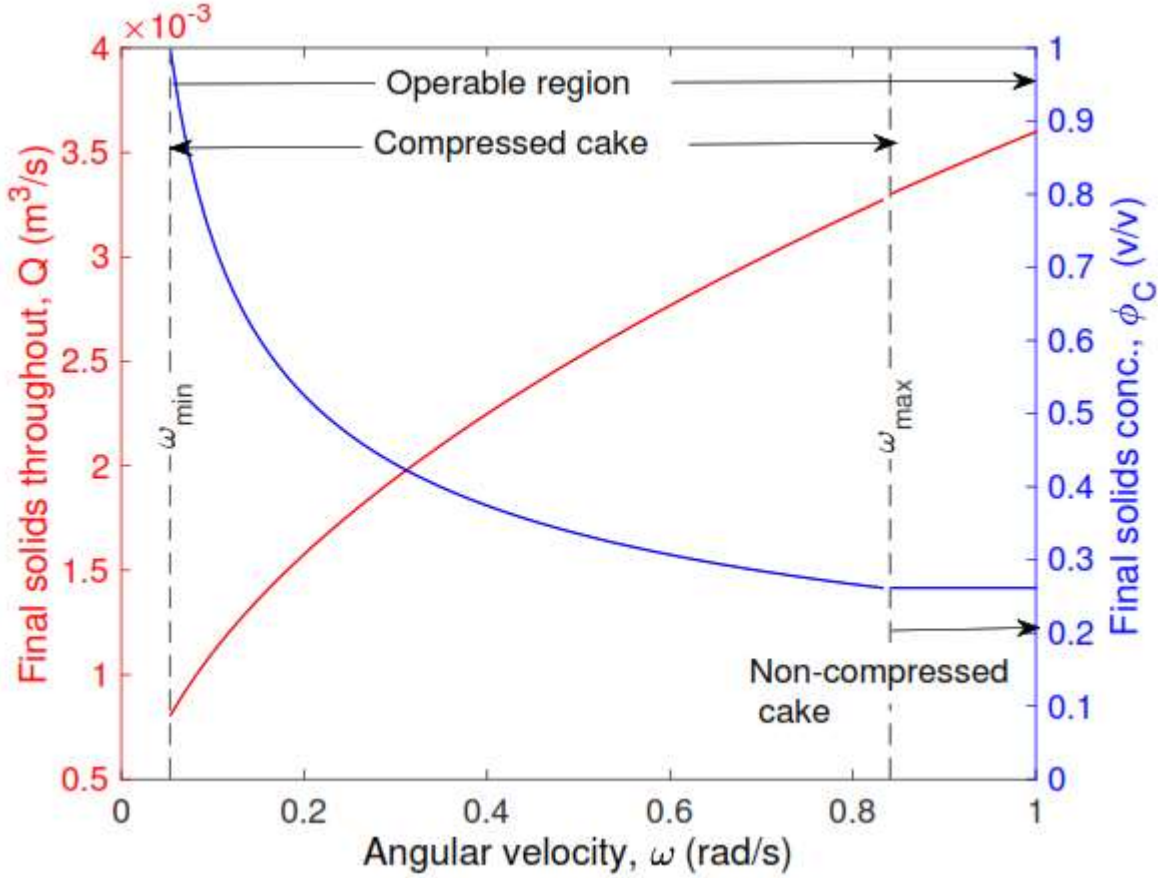


Figure 6: HPDR modelling results of solids throughput (Q) and average solids volume fraction (ϕ_c) as functions of angular velocities ($\Delta P = 10$ kPa, $\phi_0 = 0.1$ v/v, $g = 0.03$ m and $\theta_s = \pi/4$ rad).

When the angular velocity exceeds ω_{max} , only cake formation occurs until the cake exits the rolls, resulting in a fixed value of $\theta_F (= 2\pi - \theta_s)$. Consequently, the solids throughput $Q (= 2Q_F)$ changes proportionally to the square root of ω , while the solid concentration of the filter cake remains unchanged.

In any operational setting, the solids in the feed supplied to the HPDR will inevitably exhibit natural variations in the solids concentration. The variation of Q and ϕ_c with ϕ_0 is given for three vacuum pressures in Figure 7 while the angular velocity remains constant.

As the initial concentration, ϕ_0 , increases, both Q and ϕ_c increase until ϕ_c reaches its maximum designed value. Therefore, it is optimal to operate the system at the highest operable value of ϕ_0 , denoted as $\phi_{0,max}$. If ϕ_0 falls below a minimum threshold value, $\phi_{0,min}$, the HPDR will generate uncompressed filter cakes that have only undergone the cake formation stage under vacuum pressure. Further, in increasing the vacuum pressure results in a decrease of the highest operable value of ϕ_0 , which in turn narrows down the operable range for the initial feed concentration. In contrast, increasing the vacuum pressure reduces the minimum threshold value for $\phi_{0,min}$ raising the likelihood of obtaining compressed cakes even for low initial feed concentrations.

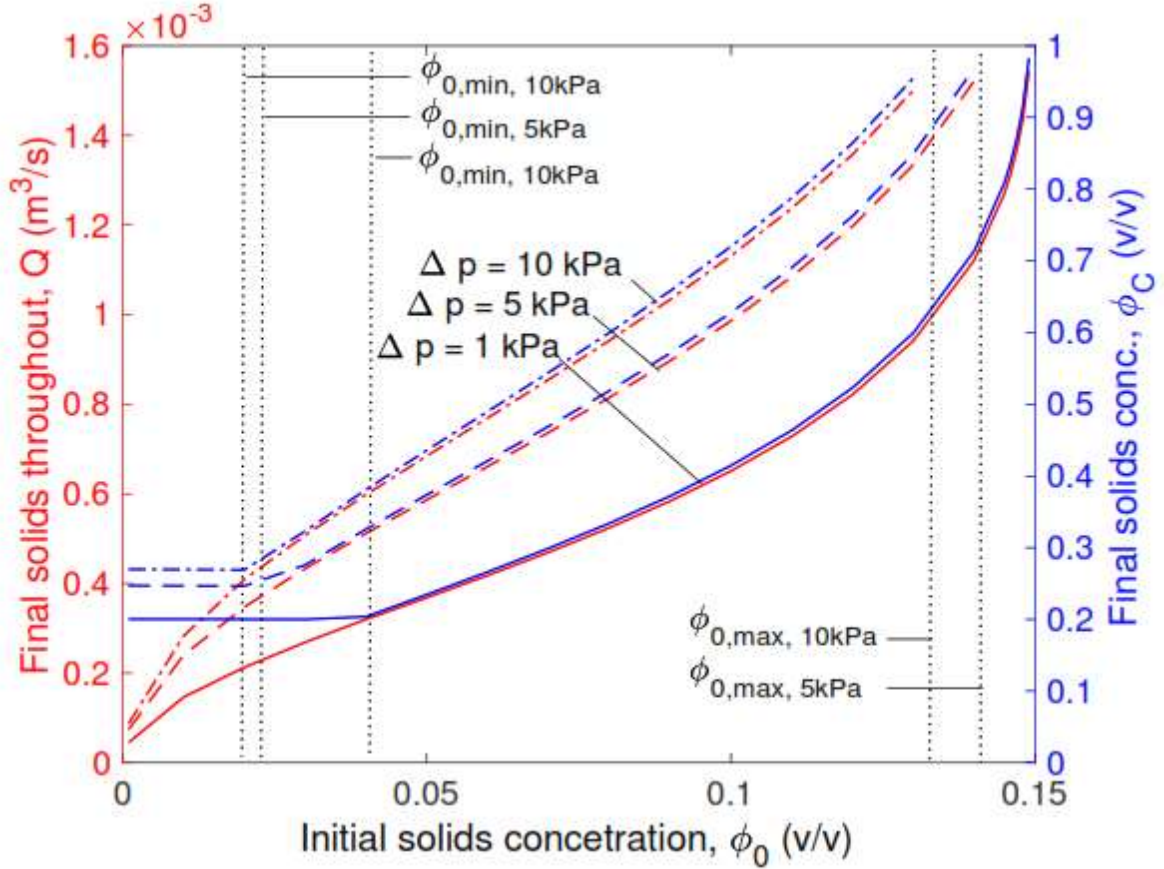


Figure 7: HPDR modelling results of solids throughput (Q) and average solids volume fraction (ϕ_c) as functions of initial solid concentration ($\Delta P = 1$ kPa, 5 kPa, 10 kPa, $\omega = 2\pi/60$ rad/s, $g = 0.03$ m and $\theta_s = \pi/4$ rad).

By examining Figure 7, it becomes possible to quantify the potential improvement in throughput and solid concentration by increasing the vacuum pressure while considering the associated cost of additional pressure. For instance, if the vacuum pressure is increased from 1 to 10 kPa at a constant initial concentration of 0.1 v/v, the throughput and solid concentration can be enhanced by 73.6% and 73.7%, respectively.

In the HPDR design stage, parameters of interest are the r/L , θ_s , and g . The variation of Q and ϕ_c with these parameters are examined for fixed operating conditions.

5. Conclusions

This study outlines the development of a one-dimensional model for the innovative filtration design known as HPDR, which was invented at the University of Melbourne and utilizes compressional rheology theory. The model involves the conservation of mass and momentum equations in one dimension, which are rearranged to derive the governing equation for pressure filtration. The model assumes two distinct stages of filtration, including cake formation under vacuum pressure followed by controlled rate cake compression. The study also presents the initial and boundary conditions for both stages, which are used to provide solutions for each process.

For the cake formation stage, an exact similarity solution analogous to vacuum-drum filtration exists under the assumptions of zero resistance of the wedge wires rolls. The formation ends, and controlled rate compression begins when the two cakes developed on the rolls meet. The meeting point of the cake is critical and depends on the operation conditions, material properties, and geometric parameters. The controlled rate compression is fully governed by geometry and some operational conditions.

Increasing the angular velocity, vacuum pressure, and initial feed concentration can enhance the final solid throughput. However, the final solid concentration of the filter cake can only be improved by elevating the latter two parameters, and increasing the angular velocity can lower the final average solid concentration. Moreover, there is a possibility that cakes will not undergo compression under low-pressure and low-initial solids or high-angular velocity conditions. Additionally, beyond certain high-pressure, high-initial solids, and low-angular velocity values, the HPDR with a fixed gap size will fail to operate. Typically, this maximum operable limit is determined by the maximum allowable and possible solid concentration based on the maximum pressure that the rolls can endure.

The numerical method established in this study enables solving the governing equations and forecasting the system's performance over various operational conditions. This method can also be utilized to optimize the HPDR's performance for a given material.

6. Notation

6.1. Roman letters

$D(\phi)$ = solids diffusivity (m^2/s)
 L = roll length (m)
 $I(\phi)$ = integral of ϕ
 p = pressure (Pa)
 $p_y(\phi)$ = compressive yield stress (Pa)
 q = filtrate flux (m/s)
 Q = solids throughput (m^3/s)
 q = suspension flux (m/s)
 $R(\phi)$ = hindered settling function ($\text{Pa}\cdot\text{s}/\text{m}^2$)
 r_r = roll radius (m)
 t = time (s)
 u = velocity (m/s)
 v = specific filtrate volume (m)
 X = similarity variable
 z = normal coordinate (m)

6.2. Greek letters

Δp = vacuum pressure (Pa)
 Δt = time step (s)
 ϕ, Φ = solids volume fraction (v/v)
 γ = similarity solution constant for $z_c(t)$ ($\text{m}/\text{s}^{1/2}$)
 η = similarity variable (m^2/s)
 η^* = similarity solution constant (m^2/s)
 Δp = vacuum pressure (Pa)
 λ = angle to hypotenuse (rad)
 θ = angle (rad)

ω = angular speed (rad/s)

6.3. Superscripts/subscripts

< = values at previous time step

0 = initial

c = cake

C = compression

f = fluid

F = formation

g = gel point

l = liquid

l = solid

max = solid

min = solid

∞ = equilibrium

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